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BEFORE THE
POSTAL REGULATORY COMMISSION
WASHINGTON, D.C. 20268-0001

PERIODIC REPORTING
(PROPOSAL ONE)

Docket No. RM2016-7

PETITION OF THE UNITED STATES POSTAL SERVICE REQUESTING INITIATION OF A PROCEEDING TO CONSIDER PROPOSED CHANGES IN ANALYTICAL PRINCIPLES (PROPOSAL ONE)
(April 5, 2016)

Pursuant to 39 C.F.R. § 3050.11, the Postal Service requests that the Commission initiate a proceeding to consider a proposal to change analytical principles relating to the Postal Service's periodic report, the Revenue Pieces and Weight (RPW) report. The proposal, labeled Proposal One, is discussed below and in greater detail in the attached text. Proposal One seeks authorization to change the current methodology for reporting revenue, pieces and weight in the RPW report for specified international outbound products.

The Postal Service requests that the Commission initiate a rulemaking proceeding pursuant to 39 C.F.R. § 3050.11 to consider this proposal.

Respectfully submitted,

UNITED STATES POSTAL SERVICE

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Proposal One

PROPOSED CHANGE IN METHODOLOGY FOR OUTBOUND RPW REPORTING (SIRVO)

OBJECTIVE:

This request seeks a change in the methodology for reporting revenue, pieces and weight for certain outbound international mail categories in the Revenue, Pieces, and Weight report (RPW). The proposed methodology and program changes relate to the *System for International Revenue and Volume, Outbound, and International Origin Destination Information System* (hereinafter referred to as SIRVO). The proposed changes will enhance the current SIRVO estimation and underlying data, and incorporate additional census data sources and census data adjustments at lower levels of detail. These changes will improve the national product estimates reflected in the RPW, and the country level estimates used by the Postal Service for monitoring business relationships, product performance, and growth opportunities. The proposed changes involve the reporting of the following subset of the outbound international RPW categories (the "Outbound Products").

- Market Dominant
    - Outbound First-Class Mail International
    - U.S. Postal Service Mail
    - Free Mail
    - International Ancillary Services
- Competitive
    - Outbound Priority Mail International

- Outbound Direct Sacks (M-bags)
- First-Class Package International Service
- International Ancillary Services

BACKGROUND:

RPW Reporting

Regulatory reporting of revenue, pieces and weight is presented in the RPW report filed quarterly with the Postal Regulatory Commission (Commission), in accordance with Commission Rule 3050.25. The RPW system used to develop this report was discussed in detail in witness Pafford's testimony (USPS-T-3) in Docket No. R2006-1. Revenue, pieces and weight data for Postal Service products are obtained through various source systems, one of which is SIRVO. SIRVO is an input into the RPW reporting process through the Adjustment RPW system (Docket No. R2006-1, Library References USPS-LR-L-18). More recently, since enactment of the Postal Accountability and Enhancement Act of 2006, SIRVO has been discussed in USPS-FY15-NP5 (FY 2015 ICRA Overview/Technical Description) Chapter 7: System for Outbound Revenue, Pieces, and Weight (SIRVO-IODIS), hereafter referred to as Chapter 7.

Current RPW Reporting of Outbound Products ("current SIRVO")

The current SIRVO estimation process was designed to provide to the RPW accurate product level estimates at the national level. The current SIRVO applies to its sample data probability-based statistical expansions to the known quarterly dispatch weights ("control weights") by destination country, mail category, mail class/subclass

(letter-post and parcel post) and receptacle type (letter trays, flat trays, bags, and bulk containers). For Canada, the expansion is also by origin exchange office.

The current RPW reporting process combines estimates developed from the current SIRVO estimation process and census data pulled from other systems as described in Chapter 7. Once the national estimates are developed, some of the national-product estimates are replaced with census data only at the national (and sometimes price-group) level. The non-replaced estimates are adjusted up or down so that the overall control weights remain constant. **Census data sources used by the current SIRVO include:**

- PostalOne!. Data from the PostalOne! system provide census data for Negotiated Service Agreements (NSA) and non-NSA mailings by product and price group. Products include International Priority Airmail (IPA), International Surface Air Lift (ISAL), Priority Mail International (PMI), Commercial ePackets, and Priority Mail Express International (PMEI). The current SIRVO product estimates are replaced by PostalOne! census data. For ISAL and IPA, this replacement occurs at the price-group level. For all other products in PostalOne!, the replacement occurs at the national product level. In addition, for IPA and Commercial ePackets, adjustments are made to the First-Class Mail International (FCMI) and First Class Package International Service (FCPIS) products that also travel in the air letter-post stream to ensure that the sum of the census, FCMI, and FCPIS product weight equals the dispatch air letter-post weight.

- Point of Sales (POS) and Click-N-Ship (CNS) systems. Data from the retail POS and CNS systems provide revenue, pieces, and weight for mail products and extra services.[1] The SIRVO product estimates are reduced by these census data to ensure that, when combined with the POS and CNS, total dispatch weight is preserved. Although these data sources have product, country, and weight-step information, the current SIRVO does not use this level of detail.

- PC Postage[2] data. PC Postage data are used by the current SIRVO at the national product level for NSA mailings of PMEI, PMI, and FCPIS. The PC Postage data are used to reduce the national product estimates so that they reflect the non-PC Postage portion of those products.

PROPOSAL:

The Postal Service would like to use the proposed SIRVO estimation and census data enhancements to estimate revenue, pieces and weight in RPW for the named outbound RPW reporting products using the new methodology, which includes much greater detail below the national and price-group level.

Proposed RPW Reporting of Outbound Products ("proposed SIRVO")

For products dispatched in the mail class letter-post streams (e.g., FCMI, FCPIS, IPA), the proposed SIRVO estimation process uses a model-based regression estimator to estimate for revenue and pieces within small sampling strata for which census weight data are available (*i.e.,* origin exchange office (OEO), destination country/country group,

[1] Currently, only insured POS and CNS transaction data are used by SIRVO.
[2] PC Postage is USPS-approved third-party vendor software that mailers can use to pay for and print their postage using a computer, printer, and internet connection.

mail category, mail class/subclass, receptacle type, year, and quarter) (hereinafter referred to as the "expansion strata"). For products dispatched in the mail class parcel post streams (e.g., PMI), where pieces are known from the dispatch system, the proposed SIRVO estimation process uses a model-based regression estimator to estimate revenue only. The difference between the use of the estimator for letter-post and parcel post flows is that the expansion strata for the parcel post regression for estimated revenue does not include OEO due to limitations on sample size.

The regression estimator provides the best linear unbiased predictor and is combined with a robust variance estimator to abate potential negative bias in the variance estimates due to model misspecifications. The estimator is used independently for each stratum, and the national-level or domain estimates are obtained through summing across relevant expansion strata level estimates. For detailed presentations of the regression estimator, please see Section 5 of Attachment A, 'Redesigned System for Outbound Revenue, Pieces and Weight (SIRVO-IODIS)' and Attachment C, 'Technical Note on SIRVO Estimator,' both attached to this pleading.

When SIRVO sample data are not available for a given expansion strata, the program looks for suitable proxy sample data beginning with the same expansion strata in an earlier time period, and then widening the criteria until usable sample data are found. For example, if no air bags from Miami to Country "X" were sampled in Miami during the reporting period, the program looks to find samples of air bags from Miami to Country "X" in an adjacent period and, if not finding any, adjusts to find air bags from all other exchange offices to Country "X" sent during that time period.

As in the current RPW reporting process, the proposed RPW reporting process would adjust the proposed SIRVO estimates with census data pulled from other systems, so that the sum of adjusted SIRVO and census equals known dispatch weight. However, in this proposal we adjust parcel post estimates to the known parcel post dispatch pieces and distribute the census data to the lowest level of detail using as much census data as currently available. The proposed SIRVO reporting process extends the use of POS, CNS, PostalOne!, and PC Postage NSA census data, and adds the use of Self-Service Kiosk (SSK) and Contract Postal Unit (CPU) census data. Section 5 of Attachment A also contains a detailed description of the adjustment process for census inputs.

Census data sources used for the proposed SIRVO include:

- PostalOne!. The PostalOne! product revenues, pieces and weight will continue to be used directly in RPW, including IPA, ISAL, Commercial ePackets, and permit-imprint PMI and PMEI. However, contrary to the current SIRVO process, PostalOne! data will be distributed to individual countries within a price group based upon the corresponding proportion of dispatch weights (if available). In addition, underlying data by NSA type in PostalOne! will be distributed proportionally to the expansion strata level.
- POS, CNS, SSK, and CPU. The POS, CNS, SSK and CPU census revenues, pieces and weight will be used directly in RPW at the country (or country group for smaller volume destination countries) and product level.
- PC Postage NSA. The PC Postage NSA census data used in the proposed approach are at the product level (not country level), like the current SIRVO.

In the future, as finer levels of data become available for PC Postage, more precise adjustments can be added to this control process.

As a result of these census adjustments, approximately 50 percent of the revenue (or 63 percent of the known dispatch weights) of the proposed SIRVO's product estimates for FY2015 are based upon census data sources. In the future, as more detailed data and additional census data sources become available, similar approaches can be used to replace proportions of the SIRVO estimates with census data.

RATIONALE:

The accuracy of outbound RPW international product and underlying country reporting can be substantially improved using the proposed SIRVO estimation and census usage approach.

IMPACT:

To facilitate the understanding and effect of these changes on the RPW report, in Attachment B to Proposal One, the Postal Service compares the FY2015 revenues, volumes and weights using the proposed SIRVO method ("Proposed" column) to the current SIRVO method ('Current' column).[3] In addition to the Proposed and Current columns, there is the amount of the change and the percent change to current method.

The first thing to notice is that FY2015 'Total All Revenue' is unchanged, while 'Total All Mail' volume increases 19.2 million pieces (0.0 percent) (page 5). Next, 'Total

[3] The hard copy public version of Attachment B is directly attached to this Proposal (with an Excel spreadsheet version attached electronically). The Postal Service is also separately filing under seal (as USPS-RM2016-7/NP1) a restricted version of Attachment B that disaggregates data pertaining to competitive products following the adopted format used in the public report. The nonpublic version filed under seal thus provides additional data for the confidential categories.

Competitive Revenue' increases $15.2 million dollars or 0.1 percent (page 4) with a corresponding decrease of $15.2 million dollars or 0.0 percent for 'Total Market Dominant Revenue (page 2). There are small changes to domestic products and services that are affected by the redistribution of international outbound revenue. The RPW reporting process ensures that the sum of product revenues 'ties out' to Accounting Trial Balance revenue.[4]

Next, we will discuss the changes to the largest international products categories. 'Outbound First-Class Mail International' or FCMI (page 1) decreases $43.7 million dollars (-13.9 percent), 31.4 million pieces (-14.8 percent), and 1.5 million pounds (-12.9 percent). 'Outbound Priority Mail International' or PMI (page 3) declines $25.4 million dollars (-4.0 percent), while volume increases 0.6 million pieces (+5.0 percent), and weight declines 0.2 million pounds (-0.3 percent). Additionally, 'First-Class Packages International Service' or FCPIS (page 3) revenue increases $16.2 million dollars (+3.6 percent), 4.4 million pieces (+15.3 percent), and 2.7 million pounds (+10.7 percent).

The impact to these outbound products reflects the changes in estimation methodology and use of census data described above. For FCMI, the average revenue per piece (AWPP) and average pound per piece (AWPP) do not change significantly. The noted declines result from a shift in the weight distributions to other products because of the lower level expansion process and finer level of census adjustments. This means that the proposed method has roughly the same mix over the rate groups and pound intervals, but the expansion process, as well as differences in the census control processes, are generating less FCMI overall.

[4] See Docket No. RM2015-9, Proposal One (June 12, 2015), Page 3, Footnote 1.

For FCPIS, the proposed method estimates slightly lighter pieces and resulting revenue per piece. This is caused by a different mix across rate groups and weight intervals.

For PMI, the proposed method estimates a slightly lighter average weight per piece and corresponding revenue per piece. Finally, the proposed method estimates a higher proportion of flat-rate envelopes.

The impacts to these outbound products reflect the changes in estimation methodology and use of census data described above.

In terms of measures of precision of the estimates, the margin of error[5] of the proposed PMI estimated package revenue is 0.4 percent and volume is zero percent.[6] This compares to the current SIRVO estimated PMI margin of error for revenue of 2.7 percent and volume of 3.6 percent. For FCMI and FCPIS traveling in the letter-post dispatch stream, the calculated FY2015 proposed margin of error for revenue is 1.0 percent and 1.2 percent for volume. The corresponding current SIRVO estimates are 3.0 percent and 2.8 percent, respectively. The precision is significantly improved under the proposed methodology.

In summary, the Proposed SIRVO approach will result in the improved reporting of outbound international revenues and volumes for the affected categories, both in terms of the level and measures of precision. With the inclusion of the improved

[5] The margin of error is calculated as 1.96 times the estimated coefficient of variation. Strictly speaking, this implies that over repeated sampling, 95 out of 100 times the confidence intervals constructed in this manner will contain the true value. Most people interpret confidence intervals as: one can be 95 percent confident the true estimate falls within plus or minus the margin of error.

[6] The proposed estimate for outbound parcels ties out to known dispatch counts. As such, there is no sampling error in the volumes.

estimator and the introduction of more census data, substantial improvements are achieved. The new system will also allow for more granularities in the estimates by country and stream, providing more information for making international product business decisions.

Attachment A

Redesigned System for Outbound Revenue, Pieces, and Weight (SIRVO-IODIS)

1. OVERVIEW

Estimates of revenue, pieces, and weight for the Postal Service's outbound international mail products are produced by the System for International Revenue and Volume, Outbound and International Origin-Destination Information System (SIRVO-IODIS). Production entails three steps. The first is to obtain international mail piece data from various postal systems. The second step is to calculate and combine data from these different systems to the desired population estimates. The last step is to report the data. These three steps are performed every Postal Quarter to produce quarterly estimates for all outbound international products.

2. OBTAINING DATA

There are several systems from which the Postal Service obtains international mail piece data. These include accounting, operation dispatching, retail and other mailing systems, product tracking, and statistical sampling systems.

There are several systems that provide revenue, pieces and weight information for all transactions ('census') for certain international mail products, such as International Priority Airmail (IPA), International Surface Air Lift (ISAL), Commercial ePacket, Priority Mail International (PMI), and Priority Mail Express International (PMEI). One system used in the production of SIRVO-IODIS estimates comes from mail entered at Bulk Mail Entry Units through postage statements. Information associated with the postage statements is keyed into *PostalOne!*, which is then accessed for producing revenue, piece and weight statistics. Point-of-Sale (POS), Click and Ship (CNS), Self-Service Kiosk (SSK), and Contract Accounting Retail System (CARS) are similar systems for retail and commercial base rate products that currently feed the reporting of revenue, pieces and weight for all types of international products.

Another source of international data is the international postal operation dispatch system called Global Business System (GBS) Dispatch. GBS Dispatch is designed to weigh every receptacle from the United States destined to a foreign country, and to apply appropriate operation identification information for transportation and billing purposes. GBS Dispatch provides census weight and, for certain classes/subclasses, census piece data by destination country, mail category transportation mode (air or SAL), mail class/subclass, receptacle type,

and origin exchange office. Census piece data are available for items dispatched under the parcel-post class (i.e., all PMI products except flat-rate envelopes and small flat-rate boxes) and certain letter-post subclasses (e.g., registered dispatched under subclass "UR", M-bags dispatched under subclass "UM", and certain tracked packet streams).

International outbound Negotiated Service Agreement (NSA) revenue, pieces and weight reports are obtained from various sources that collect census information on contracts. NSA data sources include PostalOne! for bulk mail and National Meter Account Tracking System (NMATS) for PC Postage generated contract data.

The last systems used to obtain international mail piece information are the statistical sampling systems – mainly SIRVO-IODIS, but also Origin-Destination Information System and Revenue, Pieces and Weight (ODIS-RPW). SIRVO-IODIS and ODIS-RPW provides international product information not available in other systems. SIRVO-IODIS product revenue, piece and weight estimates include First-Class Mail International (FCMI), First-Class Package International Service (FCPIS), and PMI products. ODIS-RPW provides estimates of revenue, pieces and weight for International Return Receipt, as this extra service is accurately obtainable only at the origin of entry (i.e., post office). These statistical systems are used to provide data where census data are not available and to apportion census data that are not available at the country level.

3. CALCULATING REVENUE, PIECES, AND WEIGHT

Calculating and combining international mail piece data from the various systems involves first estimating the entire population of outbound mail using SIRVO-IODIS and ODIS-RPW sample data, and then combining these estimates with the international mail piece data obtained from the product reporting systems, dispatch systems, and other sources.

Combining the sample data with census data involves four steps. First, census data that are not included in the survey data are collected by product by country. No adjustments are required for this information.

Second, survey data are adjusted so that the sum of all survey weight ties to GBS Dispatch weight. The adjustment is based on the ratio of known dispatch weight to survey weight. The weight adjustment is done by rate-change period (i.e., before and after rate change), U.S. origin exchange office, destination country (or country group for smaller volume destination countries), mail category, mail class (letter post and parcel post), and receptacle type (i.e., letter trays, flat trays, bags, and bulk containers). Collectively, these combinations of categories are known as "expansion strata." For letter-post expansion strata, the

weight ratio is also applied to survey revenue and pieces to determine dispatch-level revenue and pieces. For parcel-post expansion strata, dispatch piece counts are known, so the weight ratio is not required. Instead, a regression model is used to estimate dispatch-level revenue. The regression model estimates are developed by expansion strata grouped over all origin exchange offices. The regression model is more fully explained below. Grouping origin exchange offices across expansion strata helps to eliminate small sample size issues.

Third, any data in the survey that have corresponding data in census systems are adjusted to the levels in those census systems. Census data are disaggregated at various levels depending on the source. Most census data are specific to a country, country group, or price group. Once the census controls are applied, the resulting data are distributed across expansion strata using the SIRVO-IODIS estimates as a distribution key.

The fourth and final step is to adjust non-census-type data in the survey so that, when combined with the adjusted (step three above) census-type data in the survey, the total weight and—for certain products—pieces match the GBS Dispatch control weights and pieces by expansion strata.

4. REPORTING DATA

Reports and data files are generated after determining the international revenue, pieces and weight to the desired population totals. The data in these reports and files are used for input into the International Cost and Revenue Analysis (ICRA) report and the Annual Compliance Report (ACR).

The next three sections describe in more detail 1) SIRVO-IODIS sampling, data collection and the GBS Dispatch interface, 2) the estimation process, and 3) the process of combining product-level sample estimates with census information and official FY RPW results.

SIRVO-IODIS Sampling, Data Collection and the GBS Dispatch Interface

Design

The SIRVO-IODIS sampling scheme is a stratified random sample. Within a mail-flow stratum, a cluster sample (first stage) is selected for measurement of revenue, pieces, and weight of outbound US international mail. The reference period for this survey is a postal quarter. The clusters (known as Primary Sample Units or PSUs) within a stratum are the individual mail-flow days. A mail-flow stratum is defined the same as the expansion strata (i.e., a unique combination of US origin exchange office (OEO), destination country or country group, mail category, mail class, receptacle type). The collection of mail flow-

days makes up the SIRVO-IODIS frame. The days are the number of days in the postal quarter being considered.

The SIRVO-IODIS frame is developed by mail-flow strata and the corresponding dispatch weight sizes of mail-flow strata. Mail flows that are active (contain mail) are identified through GBS Dispatch. GBS Dispatch provides all of the mail-flow characteristics for each receptacle of mail for every outbound dispatch including weight. An average weight over a period of time is calculated from the GBS Dispatch data, which is then used to size the mail flow for the purpose of stratification. The ultimate sample units are the receptacles of mail selected from each of the PSUs.

Sample Allocation

The number of tests allocated to a mail-flow stratum depends on two constraints. First, the weight of mail moving through an OEO to the destination country is considered. Weight characteristics of all mail-flow strata are determined by analyzing the first eight weeks of the preceding postal quarter. This information is ascertained from GBS Dispatch to ensure that the strata characteristics are as current as possible. Second, the number of personnel available to sample mail at an exchange office is determined. A constrained optimum allocation algorithm determines the number of tests allocated to a stratum. The algorithm minimizes the variance for a fixed sample size while satisfying workload constraints defined by the number of data collectors available to conduct tests at each OEO processing outbound mail. The algorithm ensures that within a mail-flow stratum, at least one “mail-flow” day is scheduled for a SIRVO-IODIS test each postal accounting period (AP) (i.e., month). Some manual adjustments to mail-flow strata sample sizes are done. Beginning in FY2010, smoothing of test dates was implemented to assign a better distribution of tests per day and number of receptacles per test to match site workload. This process makes minor adjustments to the original allocation to move peak test days (days with a large number of SIRVO-IODIS tests) to days with few tests by also looking at the average number of hours it takes to record a receptacle.

Approximately two weeks before the start of each postal quarter, the SIRVO-IODIS sample is allocated, selected and sent to a data output file. The data output file of scheduled SIRVO-IODIS tests for the postal quarter is distributed to the Manager, Financial Program Compliance (MFPC). The sample includes a set of tests that GBS Dispatch will use to automatically select samples (GBS Dispatch tests) and a set of tests for most bulk containers that require manual selection of samples (non-GBS Dispatch tests).

Approximately 9,994 SIRVO-IODIS tests were scheduled in FY 2015 among the exchange offices processing outbound international mail.

Data Collection

The selection of receptacles in SIRVO-IODIS is automated (GBS Dispatch tests), except for certain bulk container tests due to dispatching time constraints (non-GBS Dispatch tests). Due to differences in bulk container processing, all bulk container tests at JFK and parcel-post bulk container tests at ORD are automatically selected; all other bulk container tests are manually selected by data collectors. GBS Dispatch tests use an algorithm built upon expected weight of the mail-flow day to select a subset of receptacles from the total number of receptacles available on the test day. The algorithm defines a weighted skip to ensure four containers are selected for the test day. The MFPC, who oversees all statistical testing within the exchange office, has the discretion to adjust this weighted skip within GBS Dispatch to ensure that the maximum numbers of receptacles are tested in the available time window. Specially trained international mail data collectors are notified by the GBS Dispatch system when receptacles have been identified for SIRVO-IODIS testing.

Data collection for non-GBS Dispatch tests on bulk mail containers follows procedures for the random selection of receptacles available for the mail-flow day. In some cases, random selection of mail pieces within selected receptacle occurs. Data collection instructions for these procedures are provided in the SIRVO-IODIS data-collection guide.

The data collector uses the Computerized On-Site Data Entry System (CODES) software on a laptop computer to record dispatch information for the sampled receptacle and the mail piece information from the selected receptacles. All mail pieces from within the selected receptacles are recorded. The recording process includes characteristics such as revenue, weight, and USPS international mail product. The data recorded from the laptop are uploaded to a secure Statistical Programs WEB base unit. The MFPC reviews and approves each test. Each month, approved tests are transferred to the mainframe computer in Eagan.

GBS Dispatch

GBS Dispatch interfaces with SIRVO-IODIS testing in the following manner. After logging onto GBS Dispatch, the dispatch clerk places a mail receptacle on the electronic scale connected to the GBS Dispatch terminal. The destination and GBS Dispatch mail category code of the receptacle are typed into the terminal. The system displays an assigned flight number, transportation legs, weight, dispatch number, sack number, class and destination. If the clerk accepts the entry, a barcode label is generated by a printer. The label is applied to a tag attached to the receptacle.

GBS Dispatch has also incorporated into its software the SIRVO-IODIS sample test schedule for GBS Dispatch tests. The system automatically knows when a particular set of dispatches is to be tested in a 24-hour period. Also, the

Statistical Programs data collector is aware of the scheduled SIRVO-IODIS test via the SIRVO-IODIS sample selection process. As the dispatch clerk places a receptacle on the electronic scale, the GBS Dispatch terminal notifies the clerk whether this receptacle is to be held for SIRVO-IODIS testing and prints out a barcode label that reflects "HOLD FOR SAMPLING". The Statistical Programs data collector is also notified via a "printer notice" sent to a printer connected to another GBS Dispatch terminal located in the sampling area that a receptacle is available for testing. Once the sample is finished, the data collector returns the receptacle into GBS Dispatch, which generates a new barcode label and removes the "hold" indicators.

5. EXPANSION AND CONTROL METHODOLOGY

Data Expansion Using A Model-Based Regression Estimator

Consider a letter-post expansion stratum for which a total of *N* receptacles are dispatched for a period. Let $Y_1, \ldots, Y_N$ be revenues from (or piece counts of) receptacles 1, …, N with their respected weights denoted by $x_1, \ldots, x_N$. The revenue or piece count for receptacle *i* is generated by the following process:

$$E(Y_i) = \beta x_i \quad \textbf{(Equation 1)}$$

$$Cov(Y_i, Y_j) = \begin{cases} \sigma^2 x_i & if\ i = j \\ 0 & if\ i \neq j \end{cases} \quad \textbf{(Equation 2)}$$

for $i = 1, \ldots, N$. β is estimated by the method of weighted least squares, and revenue (or piece count) is estimated through the model equation $\hat{\beta} x_i$ for $i = 1, \ldots, N$. Under the model, the regression estimator is reduced to the ratio estimator.

For parcel-post, both piece counts and weights are known for all receptacles. Therefore, the variable of interest Y_i is the revenue from receptacle *i* which is generated by

$$E(Y_i) = \beta_1 x_{1i} + \beta_2 x_{2i} \quad \textbf{(Equation 3)}$$

$$Cov(Y_i, Y_j) = \begin{cases} \sigma^2 x_{2i} & if\ i = j \\ 0 & if\ i \neq j \end{cases} \quad \textbf{(Equation 4)}$$

where x_{1i} and x_{2i} represent piece count and weight of receptacle *i*. The revenue of receptacle *i* is estimated by $\hat{\beta}_1 x_{1i} + \hat{\beta}_2 x_{2i}$ where regression coefficients are estimated by the weighted least squares method.

The regression estimators provide us with the best linear unbiased predictor under the specified models. Accompanying the regression estimator is a robust variance estimator, which abates potential negative bias in variance estimates due to model misspecifications.

The estimator is used independently for each expansion stratum, and the national-level or domain estimates are obtained through summing across relevant expansion strata level estimates. Please refer to Appendix C: Technical Notes on Regression Estimator for detailed descriptions of the estimator and its variance estimator.

Combining Census Data and Sample Data

The following describes in more detail the process of combining expanded sample data with census data. The process is sequential as outlined in the steps below. All adjustments are made in such a way as to maintain total GBS Dispatch weight. In this manner, the census control process does not add weight to the system or subtract weight from the system; it simply arranges it among product categories.

Step 1: Initial SIRVO Data Adjustment Using GBS Dispatch Data
For a given estimation period, unexpanded survey data from SIRVO-IODIS tests are aggregated into expansion strata. An expansion stratum is defined as a unique combination of the following elements:

- Rate-Change Period (i.e., before and after rate change)
- US Origin Exchange Office
- Destination Country or country group
- Mail Category
- Mail Class (letter-post and parcel post)
- Receptacle Type

Corresponding data from GBS Dispatch are aggregated into expansion strata. For letter-post strata, gross weight data are aggregated from GBS Dispatch. For parcel-post strata, gross weight and piece counts are aggregated from GBS Dispatch.

For each letter-post stratum, a ratio is formed based on total gross weight from GBS Dispatch divided by total unexpanded gross weight from SIRVO-IODIS tests. This ratio is used to expand gross weight, net weight, pieces, and revenue as measured in letter-post SIRVO-IODIS tests. This technique is equivalent to the process described in Equation 1 above.

For each parcel-post stratum, two ratios are formed: one for gross weight and one for piece counts. The gross weight ratio is used to expand gross weight and net weight as measure in parcel-post SIRVO-IODIS tests. The piece ratio is used to expand piece counts as measured in parcel-post SIRVO-IODIS tests.

For each parcel-post stratum, the revenue level comparable to GBS Dispatch gross weight and pieces is determined using a regression model. As described in Equation 3 above, that model regresses SIRVO-IODIS unexpanded gross weight and pieces on SIRVO-IODIS unexpanded revenue at the receptacle level for each parcel-post stratum. The model produces coefficients that are applied to GBS Dispatch gross weight and pieces to determine a corresponding revenue level, so-called "GBS Dispatch revenue." That revenue is divided by SIRVO-IODIS unexpanded revenue to form a ratio that is then used to expand the revenue measured in parcel-post SIRVO-IODIS tests.

An additional procedure included in this expansion methodology is to impute estimated values for those strata where sampling was not conducted during the estimation period. These non-sampled strata could be those where sample data were simply not collected, or they could be strata that were recently created and so were unavailable for inclusion in the sample. Imputation of the SIRVO sample data consists of using piece-level records from a similar stratum, changing only the strata variables to those of the data being imputed. By modifying only the strata variables on those piece records used for imputation, all of the variables for an individual piece of mail are retained. In a typical period, roughly 2 to 3 percent of the expansion strata are imputed (as measured by GBS Dispatch gross weight).

Step 2: M-Bags and Registered Items

M-bags are dispatched under a unique mail subclass code ("UM") for which weight and pieces are known. SIRVO-IODIS does not sample this census stream. GBS Dispatch contains data for air retail and IPA M-bags combined. A separate estimate for air retail M-bags is determined by subtracting out the level of IPA M-bags recorded in *PostalOne!*. Air retail M-bag revenue is then determined based upon published rate tables. ISAL M-bags are controlled to the level recorded in *PostalOne!*.

Registered items are also dispatched under a unique mail subclass code ("UR") for which weight and pieces are known. SIRVO-IODIS does not sample this census stream. POS provides detailed data by expansion strata and product. These data are adjusted to the GBS Dispatch levels of total pieces and weight.

Step 3: IPA PostalOne! Controls

IPA (non-M-bags) is dispatched in the air LP mailstream. The IPA portion of the air LP mailstream is controlled to *PostalOne!* totals for the same period. Controls

are applied by price group and shape. The difference in gross weight between the IPA expanded value in SIRVO-IODIS and the *PostalOne!* IPA value is proportionally shifted to all other classes in the air LP mailstream (excluding ISAL found in the air LP mailstream). This proportional adjustment is also applied to net weight, pieces, and revenue.

Step 4: ISAL PostalOne! Controls

The vast majority of ISAL (non-M-bags) is dispatched in the Surface Air Lift (SAL) LP mailstream. A small portion of ISAL is upgraded and dispatched in the air LP mailstream.

The ISAL control process is as follows. First, GBS Dispatch gross weight of SAL receptacles is used as a control of ISAL mail in the SAL mailstream. Second, *PostalOne!* gross weight is used as a control of all ISAL mail in the SAL and air mailstreams combined. In this manner, the difference between GBS Dispatch SAL and *PostalOne!* is transferred to the air mailstream so that both controls hold simultaneously. Within each price group and shape, any difference between the adjusted air portion of ISAL and the corresponding ISAL samples found in air LP portion of SIRVO-IODIS is transferred to the rest of the air LP mailstream (with the exception of IPA). A gross weight factor is used to adjust revenue, pieces, net weight, and gross weight within the air LP mailstream.

Step 5: Permit-Imprint FCMI and PMI PostalOne! Controls

Permit-imprint (PI) mailings of FCMI and PMI are recorded in *PostalOne!*. These mailings are dispatched in the air letter-post and air parcel-post mailstreams and included in SIRVO-IODIS sampling. All sample data for these mailings are controlled to *PostalOne!* by rate-change period, mail product, price group, destination country/country group, label class, exchange office, receptacle type, and shape. Within each of these groupings, the difference in gross and net weight is moved from the PI indicia group to non-PI indicia group (stamp, meter, PVI, IBI, etc.), so that the overall dispatch weight remains unchanged. For the parcel-post label class, this same technique is applied to piece totals. That is, *PostalOne!* piece counts replace SIRVO-IODIS piece counts, and any difference is shifted to the SIRVO-IODIS non-PI stream so that overall dispatch piece totals remain unchanged. For the letter-post label class, SIRVO-IODIS pieces-per-net-pound factors are developed by grouping. These factors are applied to the adjusted non-PI net pounds to impute non-PI pieces. Likewise, for both of the letter-post and parcel-post streams, non-PI revenue is imputed based on SIRVO-IODIS revenue-per-piece factors.

Step 6: EDW Controls

Data from POS, CNS, SSK, and CARS census systems are aggregated in the Enterprise Data Warehouse (EDW) data system. Census data are available by product and destination country/country group. As with the *PostalOne!* controls

described above, the EDW adjustments are made in such a way as to maintain total GBS Dispatch gross weight.

The control process is sequential.[1] First, SIRVO-IODIS data for mail pieces containing a Postal Validation Imprint (PVI) indicia are scaled to POS totals in EDW. The differences between the original (pre-scaled) values and the controlled values are shifted to all corresponding non-POS mail pieces. Then, from the remaining pool of non-POS mail pieces, CNS pieces are isolated and scaled to CNS totals in EDW. The differences between the original values and the controlled values are shifted to all corresponding non-CNS mail pieces. Then, from the remaining pool of non-CNS mail pieces, SSK pieces are isolated and controlled in the same manner. Finally, using the remaining pool of non-SSK pieces, CARS pieces are isolated and controlled in the same manner.

Step 7: PC Postage NSA Controls

This procedure compares SIRVO-IODIS PMI variable rate and FCPIS commercial plus IBI expanded sample data for PC Postage NSA vendors to comparable census data. At the present time, census data are only available at a national level. SIRVO-IODIS sample weight is replaced with corresponding census weight for each PC Postage vendor. In order to not change the overall level of PMI variable rate weight and FCPIS weight, any residuals between the two systems are moved first to the corresponding levels of pieces containing a commercial base marking and then, if that is exhausted, to pieces containing no marking. The end result is that when census weight is added to the adjusted SIRVO-IODIS weight, the overall level of PMI and FCPIS remains unchanged. Pieces and revenue are adjusted using SIRVO-IODIS per-piece factors.

Step 8: Insured PMEI and PMI

POS, CNS, SSK, and CARS source data are used directly in the RPW for insured transactions for extra services.

[1] The order of the sequence does not affect the final results. In addition, equivalent results occur if all controls are applied simultaneously.

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MARKET DOMINANT PRODUCTS
FINAL REVENUE, PIECES, AND WEIGHT BY CLASSES OF MAIL AND SPECIAL SERVICES FOR
FISCAL YEAR 2015 (Oct. 1, 2014-Sep. 30, 2015)
(Data in Thousands)

Service Category	REVENUE				PIECES				WEIGHT (Pounds)			
			Change Proposed over Current				Change Proposed over Current				Change Proposed over Current	
	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent
First-Class Mail:												
Single-Piece Letters	9,973,401	9,956,080	17,321	0.2	19,804,675	19,771,946	32,730	0.2	604,969	603,814	1,155	0.2
Single-Piece Cards	299,429	298,385	1,044	0.3	847,309	844,316	2,993	0.4	5,410	5,392	19	0.3
Total Single-Piece Letters and Cards	10,272,830	10,254,465	18,364	0.2	20,651,984	20,616,262	35,723	0.2	610,379	609,205	1,174	0.2
Presort Letters	14,981,115	14,981,115	0	0.0	38,004,707	38,004,707	0	0.0	2,193,024	2,193,024	0	0.0
Presort Cards	569,486	569,486	0	0.0	2,169,537	2,169,537	0	0.0	17,715	17,715	0	0.0
Total Presort Letters and Cards	15,550,601	15,550,601	0	0.0	40,174,244	40,174,244	0	0.0	2,210,739	2,210,739	0	0.0
Flats	2,372,320	2,366,527	5,794	0.2	1,677,326	1,673,933	3,393	0.2	341,562	340,772	791	0.2
Parcels	546,449	545,505	944	0.2	200,741	200,387	354	0.2	61,937	61,827	110	0.2
Domestic Negotiated Serv. Agreement Mail	0	0	0	-	0	0	0	-	0	0	0	-
Outbound First-Class Mail International	271,101	314,797	(43,697)	(13.9)	180,777	212,184	(31,407)	(14.8)	9,927	11,394	(1,466)	(12.9)
Inbound Intl. Letter-Post Single-Piece & NSA Mail	426,462	426,462	0	0.0	473,600	473,600	0	0.0	154,926	154,926	0	0.0
First-Class Mail Fees	148,025	148,007	18	0.0	0	0	-	-	0	0	-	-
First-Class Dom. NSA Mail Fees	0	0	-	-	0	0	-	-	0	0	-	-
Total First-Class Mail	29,587,788	29,606,364	(18,577)	(0.1)	63,358,671	63,350,609	8,062	0.0	3,389,470	3,388,862	608	0.0
Standard Mail:												
High Density and Saturation Letters	991,349	991,349	0	0.0	6,478,281	6,478,281	0	0.0	263,618	263,618	0	0.0
High Density and Saturation Flats & Parcels	2,036,738	2,036,738	0	0.0	11,232,313	11,232,313	0	0.0	2,024,627	2,024,627	0	0.0
Carrier Route	2,237,300	2,237,300	0	0.0	8,291,048	8,291,048	0	0.0	1,794,312	1,794,312	0	0.0
Letters	10,022,982	10,022,982	0	0.0	47,720,675	47,720,675	0	0.0	2,397,007	2,397,007	0	0.0
Flats	2,106,149	2,106,149	0	0.0	5,248,505	5,248,505	0	0.0	1,344,776	1,344,776	0	0.0
Parcels	65,093	65,093	0	0.0	60,420	60,420	0	0.0	22,646	22,646	0	0.0
Every Door Direct Mail Retail	148,385	148,385	0	0.0	832,566	832,566	0	0.0	105,602	105,602	0	0.0
Domestic Negotiated Serv. Agreement Mail	53,217	53,217	0	0.0	226,464	226,464	0	0.0	44,709	44,709	0	0.0
Inbound Intl. Negotiated Serv. Agreement Mail	0	0	0	-	0	0	0	-	0	0	0	-
Standard Mail Fees	50,081	50,058	23	0.0	0	0	-	-	0	0	-	-
Standard Mail Dom. NSA Mail Fees	0	0	-	-	0	0	-	-	0	0	-	-
Total Standard Mail	17,711,295	17,711,272	23	0.0	80,090,273	80,090,273	0	0.0	7,997,298	7,997,298	0	0.0
Periodicals Mail:												
In-County	66,015	66,015	0	0.0	570,817	570,817	0	0.0	163,941	163,941	0	0.0
Outside County	1,515,354	1,515,354	0	0.0	5,267,358	5,267,358	0	0.0	2,003,821	2,003,821	0	0.0
Periodicals Mail Fees	7,854	7,853	1	0.0	0	0	-	-	0	0	-	-
Total Periodicals Mail	1,589,223	1,589,222	1	0.0	5,838,175	5,838,175	0	0.0	2,167,762	2,167,762	0	0.0
Package Services Mail:												
Alaska Bypass	33,762	33,762	0	0.0	1,282	1,282	0	0.0	86,474	86,474	0	0.0
Inbound Intl. Surface Parcel Post (at UPU Rates)	0	0	-	-	0	0	-	-	0	0	-	-
Inbound Intl. Negotiated Service Agreement Mail	0	0	-	-	0	0	-	-	0	0	-	-
Bound Printed Matter Flats	212,356	212,356	0	0.0	260,492	260,492	0	0.0	411,428	411,428	0	0.0
Bound Printed Matter Parcels	283,462	283,462	0	0.0	227,911	227,911	0	0.0	553,967	553,967	0	0.0
Media and Library Mail	276,456	275,781	676	0.2	75,433	75,242	191	0.3	186,286	185,882	404	0.2
Package Services Mail Fees	2,770	2,768	3	0.1	0	0	-	-	0	0	-	-
Total Package Services Mail	808,806	808,128	678	0.1	565,118	564,927	191	0.0	1,238,155	1,237,751	404	0.0

Public

MARKET DOMINANT PRODUCTS
FINAL REVENUE, PIECES, AND WEIGHT BY CLASSES OF MAIL AND SPECIAL SERVICES FOR
FISCAL YEAR 2015 (Oct. 1, 2014-Sep. 30, 2015)
(Data in Thousands)

Service Category	REVENUE				PIECES				WEIGHT (Pounds)			
			Change Proposed over Current				Change Proposed over Current				Change Proposed over Current	
	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent
U.S. Postal Service Mail	0	-	-	-	362,954	361,420	1,534	0.4	114,723	114,201	522	0.5
Free Mail	0	-	-	-	45,748	45,508	240	0.5	18,594	18,458	136	0.7
Total Market Dominant Mail	49,697,112	49,714,986	(17,874)	(0.0)	150,260,939	150,250,912	10,027	0.0	14,926,001	14,924,331	1,670	0.0
Ancillary Services:												
Certified Mail	664,930	662,734	2,196	0.3	198,464	197,810	654	0.3				
Collect on Delivery	3,116	3,113	3	0.1	319	319	0	0.1				
Delivery Confirmation	40,591	40,591	0	0.0	3,550,053	3,550,053	0	0.0				
Insurance	85,250	85,242	8	0.0	17,233	17,230	3	0.0				
Registered Mail	35,945	35,925	20	0.1	2,293	2,291	2	0.1				
Return Receipts	372,027	370,957	1,070	0.3	156,943	156,552	391	0.2				
Stamped Envelopes and Cards	12,449	12,449	0	0.0	0	0	-	-				
Other Domestic Ancillary Services	78,200	77,968	232	0.3	29,793	29,704	89	0.3				
International Ancillary Services	42,241	43,084	(843)	(2.0)	29,046	29,108	(61)	(0.2)				
Total Ancilliary Services	1,334,749	1,332,063	2,686	0.2	3,984,145	3,983,068	1,077	0.0				
Special Services:												
Money Orders	158,856	158,856	0	0.0	92,776	92,776	0	0.0				
Post Office Box Service	310,928	310,928	0	0.0	6,016	6,016	0	0.0				
Other Domestic Special Services	117,266	117,266	0	0.0	2,274	2,274	0	0.0				
Other International Special Services	9	9	0	0.0	914	914	0	0.0				
Total Additional Special Services	587,059	587,059	0	0.0	101,981	101,981	0	0.0				
Total Market Dominant Services	1,921,809	1,919,123	2,686	0.1	4,086,126	4,085,049	1,077	0.0				
Total Market Dominant Mail and Services	51,618,921	51,634,108	(15,188)	(0.0)								
Other Market Dominant Revenue 4/	798,198	798,198	0	0.0								
Total Market Dominant Revenue	52,417,119	52,432,307	(15,188)	(0.0)								

Public

COMPETITIVE PRODUCTS
FINAL REVENUE, PIECES, AND WEIGHT BY CLASSES OF MAIL AND SPECIAL SERVICES FOR
FISCAL YEAR 2015 (Oct. 1, 2014-Sep. 30, 2015)
(Data in Thousands)

Service Category	REVENUE				PIECES				WEIGHT (Pounds)			
			Change Proposed over Current				Change Proposed over Current				Change Proposed over Current	
	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent
Express Mail:												
Total Express Mail	779,067	779,067	0	0.0	35,705	35,705	0	0.0	38,421	38,421	0	0.0
First-Class Package Service:												
Total First Class Package Service	1,735,301	1,730,795	4,505	0.3	728,958	727,079	1,879	0.3	256,574	255,912	662	0.3
Standard Post Mail:												
Total Standard Post	474,822	474,586	236	0.0	32,531	32,509	22	0.1	187,589	187,494	96	0.1
Priority Mail:												
Total Priority Mail	7,241,641	7,226,171	15,470	0.2	984,876	982,563	2,313	0.2	2,213,457	2,209,013	4,444	0.2
Parcel Select Mail:												
Total Parcel Select Mail	3,298,476	3,298,476	0	0.0	1,876,634	1,876,634	0	0.0	3,578,361	3,578,361	0	0.0
Parcel Return Service Mail:												
Total Parcel Return Service Mail	152,301	152,301	0	0.0	59,610	59,610	0	0.0	170,763	170,763	0	0.0
International Mail:												
Outbound Priority Mail International	617,354	642,760	(25,406)	(4.0)	12,331	11,748	582	5.0	82,090	82,331	(241)	(0.3)
Outbound International Expedited Services	247,639	247,639	0	0.0	3,937	3,937	0	0.0	24,014	24,014	0	0.0
Other Outbound International Mail	863,799	847,794	16,004	1.9	227,383	222,989	4,394	2.0	86,290	83,582	2,709	3.2
Inbound International	234,295	234,295	0	0.0	15,331	15,331	0	0.0	103,445	103,445	(0)	(0.0)
International Mail Fees	155	155	1	0.3	0	0	-	-	0	0	-	-
Total International Mail	1,963,243	1,972,643	(9,401)	(0.5)	258,982	254,005	4,976	2.0	295,839	293,371	2,468	0.8
Total Competitive Mail	15,644,851	15,634,040	10,810	0.1	3,977,296	3,968,105	9,191	0.2	6,741,004	6,733,334	7,670	0.1

Public

COMPETITIVE PRODUCTS
FINAL REVENUE, PIECES, AND WEIGHT BY CLASSES OF MAIL AND SPECIAL SERVICES FOR
FISCAL YEAR 2015 (Oct. 1, 2014-Sep. 30, 2015)
(Data in Thousands)

	REVENUE				PIECES				WEIGHT (Pounds)			
			Change Proposed over Current				Change Proposed over Current				Change Proposed over Current	
Service Category	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent
Ancillary Services:												
Other Domestic Ancillary Services	3,919	3,902	17	0.4	685	681	3	0.5				
International Ancillary Services	10,728	6,367	4,360	68.5	1,874	1,556	317	20.4				
Total Ancilliary Services	14,646	10,269	4,377	42.6	2,558	2,238	321	14.3				
Special Services:												
Premium Forwarding Service	24,443	24,443	0	0.0	1,321	1,321	0	0.0				
Intl. Money Orders & Money Transfer Service	1,000	1,000	0	0.0	141	141	0	0.0				
Other Domestic Special Services	750,073	750,073	0	0.0	86,145	86,145	0	0.0				
Other International Special Services	0	0	-	-	0	0	-	-				
Total Special Services	775,516	775,516	0	0.0	87,607	87,607	0	0.0				
Total Competitive Services	790,162	785,785	4,377	0.6	90,165	89,845	321	0.4				
Total Competitive Mail and Services	16,435,013	16,419,825	15,188	0.1								
Other Competitive Revenue	99,049	99,049	0	0.0								
Total Competitive Revenue	16,534,062	16,518,875	15,188	0.1								

Attachment B - Public

Public

TOTAL MARKET DOMINANT AND COMPETITIVE PRODUCTS
FINAL REVENUE, PIECES, AND WEIGHT BY CLASSES OF MAIL AND SPECIAL SERVICES FOR
FISCAL YEAR 2015 (Oct. 1, 2014-Sep. 30, 2015)
(Data in Thousands)

Service Category	REVENUE				PIECES				WEIGHT (Pounds)			
			Change Proposed over Current				Change Proposed over Current				Change Proposed over Current	
	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent	Proposed	Current	Amount	Percent
Total Market Dominant and Competitive												
Total All Mail	65,341,962	65,349,026	(7,063)	(0.0)	154,238,235	154,219,017	19,218	0.0	21,667,005	21,657,666	9,340	0.0
Total All Services	2,711,971	2,704,908	7,063	0.3	4,176,291	4,174,893	1,398	0.0				
Total All Mail and Services	68,053,934	68,053,934	(0)	(0.0)								
Total All Other Revenue	897,248	897,248	0	0.0								
Total All Revenue	68,951,181	68,951,181	(0)	(0.0)								

- Report totals may not sum due to rounding.

Appendix: Technical Note on SIRVO Estimator

1 Preliminary

1.1 Objective

The objective of this technical note is to describe the estimation methodology used for SIRVO.

1.2 Notation and Model

SIRVO takes the model-based approach. The numbers $y_1, \ldots, y_N$ are treated as realized values of random variables $Y_1, \ldots, Y_N$ that are assumed to be generated by a statistical model. Throughout the tech note, Y_i denotes a "random response variable" of item i, e.g., revenue from receptacle i.

Let $\boldsymbol{Y}$ be a $N \times 1$ random response vector, e.g., revenue, and $\boldsymbol{X}$ a $N \times p$ design matrix where N is the number of receptacles and p the number of auxiliary variables. The model for both letter and parcel posts is expressed as

$$E(\mathbf{Y}) = \mathbf{X}\boldsymbol{\beta} \tag{1.1a}$$
$$V(\mathbf{Y}) = \sigma^2 \mathbf{W} \tag{1.1b}$$

where $\boldsymbol{\beta}$ is a $p \times 1$ regression coefficients, $\boldsymbol{W}$ a known diagonal covariance matrix, and σ^2 an unknown constant. A random sample of n receptacles is selected and a response y_i is measured for each sample unit. Without loss of generality, we arrange the response vector $\boldsymbol{Y}$ so that the first n elements in $\boldsymbol{Y}$ correspond to the sample units and partition $\boldsymbol{Y}$, $\boldsymbol{X}$, and $\boldsymbol{W}$ to sample and non-sample units:

$$\boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}_1 \\ \boldsymbol{Y}_2 \end{pmatrix}, \boldsymbol{X} = \begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{pmatrix}, \boldsymbol{W} = \begin{pmatrix} \boldsymbol{W}_1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{W}_2 \end{pmatrix}$$

therefore, $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ are $n \times 1$ and $(N-n) \times 1$ vectors, and the dimensions of $\boldsymbol{X}_1$, $\boldsymbol{X}_2$, $\boldsymbol{W}_1$, and $\boldsymbol{W}_2$ are $n \times p$, $(N-n) \times p$, $n \times n$, and $(N-n) \times (N-n)$, respectively.

2 Predictor of Totals

2.1 Regression Estimator

Let $\mathbf{1}_1$ and $\mathbf{1}_2$ be vectors of the form $(1, 1, \cdots, 1)^t$ with lengths n and $(N-n)$, respsectively. Superscript "t" indicates the transpose of a vector or a matrix. A model unbiased predictor of a total $T = \mathbf{1}_1^t \boldsymbol{Y}_1 + \mathbf{1}_2^t \boldsymbol{Y}_2$ is

$$\hat{T} = \mathbf{1}_1^t \boldsymbol{Y}_1 + \mathbf{1}_2^t \boldsymbol{X}_2 \hat{\boldsymbol{\beta}} \tag{2.1}$$

where $\hat{\boldsymbol{\beta}} = \boldsymbol{A}^{-1} \boldsymbol{X}_1^t \boldsymbol{W}_1^{-1} \boldsymbol{Y}_1$ with $\boldsymbol{A} = \boldsymbol{X}_1^t \boldsymbol{W}_1^{-1} \boldsymbol{X}_1$. $\hat{\boldsymbol{\beta}}$ is the best linear unbiased estimator of $\boldsymbol{\beta}$ which consequently makes $\hat{T}$ the best linear unbiased predictor (BLUP) of T under (1.1a) and (1.1b).

2.1.1 Letter Post

For first-class international cards, letters, flats, packages, and priority mail products that are dispatched with first class products, e.g., priority mail international flat rate envelopes, the sole auxiliary variable is the "receptacle weight," therefore, the dimensions of the design matrix $\boldsymbol{X}$ are $N \times 1$. We assume that $V(\boldsymbol{Y}) = \sigma^2 \mathbf{diag}\{\boldsymbol{X}\}$ where **diag** is a diagonal matrix with diagonal entries expressed inside the brace. The regression estimator with this specification is reduced to the ratio estimator.

2.1.2 Parcel Post

For priority mail international products that are considered parcel post, piece counts and weights are available for all receptacles, and an $N \times 2$ design matrix $\boldsymbol{X}$ is formed. The variance of $\boldsymbol{Y}$ is assumed to be the same as that of letter post.

2.2 Strata-Level Prediction

Let h be an index for strata and be defined over a set $\{1, \ldots, H\}$ where H is the number of strata. The regression estimator described in section 2.1 is independently applied to each stratum. To predict a total for stratum h, denoted by T_h, the regression predictor is

$$\hat{T}_h = \mathbf{1}_1^t \boldsymbol{Y}_{h1} + \mathbf{1}_2^t \boldsymbol{X}_{h2} \hat{\boldsymbol{\beta}}_h \tag{2.2}$$

where subscripts h simply indicate that the predictor based on the sample taken for stratum $h \in \{1, \ldots, H\}$.

2.3 Domain-Level Prediction

For a given domain (subpopulation) D comprising multiple strata, the predictor of domain total T_D, e.g., country-level total, is

$$\hat{T}_D = \sum_{h \in D} \hat{T}_h$$

3 Variance Estimator

3.1 Robust Variance Estimator

From (2.2), the prediction error is expressed as

$$\hat{T} - T = \mathbf{1}_2^t \boldsymbol{X}_2 \hat{\boldsymbol{\beta}} - \mathbf{1}_2^t \boldsymbol{Y}_2$$

Then the prediction error variance of $\hat{T}$ becomes

$$V\left(\hat{T} - T\right) = \sigma^2 \left(\mathbf{1}_2^t \boldsymbol{X}_2 \boldsymbol{A}^{-1} \boldsymbol{X}_2^t \mathbf{1}_2 + \mathbf{1}_2^t \boldsymbol{W}_2 \mathbf{1}_2\right) \tag{3.1}$$

which can be estimated by

$$\hat{V}\left(\hat{T} - T\right) = \hat{\sigma}^2 \left(\mathbf{1}_2^t \boldsymbol{X}_2 \boldsymbol{A}^{-1} \boldsymbol{X}_2^t \mathbf{1}_2 + \mathbf{1}_2^t \boldsymbol{W}_2 \mathbf{1}_2\right)$$

where

$$\hat{\sigma}^2 = \frac{1}{n-p} \left(\boldsymbol{Y}_1 - \boldsymbol{X}_1 \boldsymbol{\beta}\right)^t \boldsymbol{W}_1^{-1} \left(\boldsymbol{Y}_1 - \boldsymbol{X}_1 \boldsymbol{\beta}\right)$$

$\hat{V}(\hat{T} - T)$ is unbiased for (3.1) if the model assumptions (1.1a) and (1.1b) hold. Deviations from (1.1a) and/or (1.1b) make $\hat{V}(\hat{T} - T)$ severely underestimate the true error variance (3.1).

We can, however, construct a variance estimator that is robust against deviations from model assumptions (1.1a) and (1.1b). We first rewrite the prediction error as

$$\hat{T} - T = \boldsymbol{c}^t \boldsymbol{Y}_1 - \mathbf{1}_2^t \boldsymbol{Y}_2$$

with $\boldsymbol{c}^t = \mathbf{1}_2^t \boldsymbol{X}_2 \boldsymbol{A}^{-1} \boldsymbol{X}_1^t \boldsymbol{W}_1^{-1}$ and assume that the true covariance matrix is $\boldsymbol{V}$ with v_i representing ith diagonal entry of $\boldsymbol{V}$. Then we have

$$V\left(\hat{T} - T\right) = \boldsymbol{c}^t \boldsymbol{V}_1 \boldsymbol{c} + \mathbf{1}_2^t \boldsymbol{V}_2 \mathbf{1}_2 \tag{3.2}$$

We now use the fact that

$$E\left(y_i - \boldsymbol{x}_i^t\boldsymbol{\beta}\right)^2 = v_i\left(1 - p_i\right) \tag{3.3}$$

where p_i is the ith diagonal element of the projection matrix $\boldsymbol{P} = \boldsymbol{W}^{-1}\boldsymbol{X}_1\boldsymbol{A}^{-1}\boldsymbol{X}_1$, i.e., $p_i = w_i^{-1}\boldsymbol{x}_i\boldsymbol{A}^{-1}\boldsymbol{x}_i$ with $\boldsymbol{x}_i$ being ith row of $\boldsymbol{X}$, i.e., $(x_{i1}, x_{i2}, \ldots, x_{ip})$. Since $p_i < 1$, we see that the squared error in (3.3) tends to underestimate the true variance v_i, but this negative bias could be easily adjusted by scaling the both sides of (3.3) by $(1-p_i)^{-1}$. The above argument suggests that we obtain a robust variance estimator of (3.2) through the correction term $(1 - h_i)^{-1}$,

$$\begin{aligned}\hat{V}_{\text{robust}}\left(\hat{T} - T\right) = \boldsymbol{c}^t\mathbf{diag}\left\{\left(y_i - \boldsymbol{x}_i^t\hat{\boldsymbol{\beta}}\right)^2\left(1-p_i\right)^{-1}\right\}\boldsymbol{c} \\ + \left(\mathbf{1}_2^t\boldsymbol{W}_2\mathbf{1}_2\right)\frac{\mathbf{1}_1^t\mathbf{diag}\left\{\left(y_i - \boldsymbol{x}_i^t\hat{\boldsymbol{\beta}}\right)^2\left(1-p_i\right)^{-1}\right\}\mathbf{1}_1}{\mathbf{1}_1^t\boldsymbol{W}_1\mathbf{1}_1}\end{aligned} \tag{3.4}$$

where **diag** is a diagonal matrix with diagonal entries expressed inside the brace.

The robust variance estimator (3.4) is applied to each stratum $h \in \{1, \ldots, H\}$, and we obtain the error variance estimate $\hat{V}_{\text{robust},h}(\hat{T}_h - T_h)$. Then the variance estimate for a domain total $\hat{T}_D$ can be obtained by

$$\hat{V}_{\text{robust}}\left(\hat{T}_D - T_D\right) = \sum_{h\in D}\hat{V}_{\text{robust},h}\left(\hat{T} - T\right)$$